Redoing the Numbers

Secondary Math

for a Postsecondary Work World

John Woodward

It is a quiet, but recurrent question at any number of conferences on students with disabilities: "How will our students meet the challenges of a changing work force?" Even though this concern arises for many of our students in public schools, it is an especially provocative question for students with learning disabilities. Longitudinal data collected over the 1980s on these students and their employment history after leaving school are far from comforting: Postsecondary employment for students with learning disabilities is likely to become increasingly one of low-wage, service industry jobs. For students like Leona (see box, "Leona's Story"), failure in mathematics probably means far fewer employment options in the future.

The situation isn't any easier for Leona's teacher, Terry Wilson. Like many remedial and special education teachers, he's inclined to "look back" and concentrate on the skills his students haven't yet mastered. He and the two other remedial math teachers in Marshall Junior High use the same highly sequenced, basic skills curriculums and supplementary worksheets. These materials have been used for years at the school. But Terry Wilson is also aware that practice on isolated skills does little to prepare his remedial students for what he demands of more capable students in his three algebra classes. Besides, the notion of work-

Leona's Story

Put yourself in the position of Leona, a ninth grader at Marshall Junior High School. For years, you have been frustrated with math, and you've failed. It's been discouraging to see many of your friends do better than you, only to move on to more advanced math classes. Math seems like an endless repetition of the "same old thing." It's "baby stuff," and you've "seen it so many times before, what's the point?" On other occasions, particularly in your mainstreamed remedial math class, it moves much too quickly and you just don't understand what the teacher is talking about.

Students like Leona experience a holding pattern when it comes to the math that follows the basic operations these students learned in elementary school. Fractions, decimals, percents, and negative numbers are confusing. Teachers tell them they're important and that students need to know them to take algebra. But for students with learning disabilities in mathematics, these concepts are hard to visualize, and they don't seem to apply to anything they do or see in everyday life.

There is no less of a problem for Terry Wilson, Leona's mainstreamed Math Apps teacher. He encounters a range of ability differences in this low-track math class that is greater than anything he faces all day. Some students in the class need "brush up" work: a little review and some concentrated practice on a few topics such as decimals. Others still can't remember their multiplication facts; and many students make a range of minor, but critical, mistakes when performing multiplication or division on large numbers. Wilson dreads the kind of review and practice it takes to teach these students "one more time," and he has little confidence that such practice will permanently solve the problem. His lowest-performing students, most of whom have individualized education programs (IEPs) for mathematics, typically regress back to bad habits after a couple of weeks. And that assumes that he can motivate these students enough to take his worksheet practice seriously. Wilson would like to take his most challenging students beyond the rote "drill and kill," but he doesn't know how.

"How will our students meet the challenges of a changing work force?"

ing on a student's individual skill deficits sounds sensible in the abstract, but as "Leona's Story" suggests, the range of unmastered skills is overwhelming when there are 20 students in the class.

Wilson is also aware that "looking back" and focusing on discrete skills such as long division or decimal multiplication ignores a world that is only 3 years away (at most) for his students. He is fully aware that this world is filled with all kinds of technological devices that do the long division and multiply the decimals. What his students need has been said in many documents about the future work force over the past 10 years. They need to learn how to manage information, communicate with others, and use technology appropriately. They need to know math, but they need to know much more.

The Workplace Literacy Project

For the past 2 years, Terry Wilson and his class have participated in a pilot project funded by the U.S. Department of Education, Office of Special Education Programs, in conjunction with support from the Microsoft Corporation. The project addressed some key work force literacy issues by using common technologies such as calculators, Microsoft Word, and Excel as integral parts of mathematics and writing. Rather than concentrating on applied problems only—a tactic commonly found in high school consumer math courses—the project blended conceptual approaches to mathematics with problem-solving exercises that were anchored in the students' world. This last element was crucial, because students learned to solve problems that they readily understood, and they did so by collecting and analyzing real data. This was a time-consuming approach, and it would have been impossible were it not for a dramatic reduction in the time traditionally devoted to learning and mastering paper-and-pencil skills. Freeing students from countless practice worksheets afforded a greater emphasis on conceptual understanding, application, and varied forms of written and oral communication.

The way Leona finally "learned" fractions is a good example of how the Workplace Literacy Project moved quickly from traditional exercises to a more conceptual approach to math, and one that integrated technology. Wilson began the unit on fractions by using a variety of visual representations (e.g., pie diagrams, number lines, measuring cups). Gradually, he linked these pictorial representations to symbols. He also spent considerable time discussing with his students how fractions differed from whole numbers. In fact, students were encouraged to construct and even diagram comparisons of fractions to "other kinds of numbers." Initially, one student concluded that the smallest possible fraction was 1/16 because, "you can't draw any more lines in the circle [pie diagram], so you can't get any smaller pieces."

Students both with and without disabilities need to learn how to manage information, communicate with others, and use technology appropriately. They need to know math, but they need to know much more.

Incorrect as this observation may have been, Wilson was pleased that these students were beginning to talk about math for the first time in years. Wilson found it a refreshing change from the sighs, grunts, and one-word answers in his past classes. It also answered one of the greatest challenges he had always had in remedial classes: breaking students of the habit of either not talking, feeling ashamed if they have the "wrong" answer, or glibly dismissing assignments as boring or baby work. Over time, Wilson introduced problems that involved adding and subtracting fractions with the same denominator, as well as multiplying fractions with different denominators.

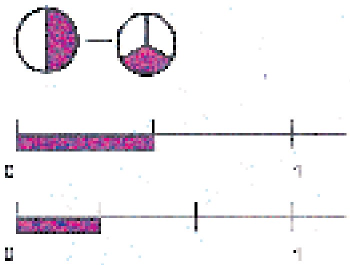
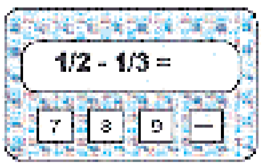
Technology and Conceptual Understanding

Rather than continue working slowly through step-by-step worksheets that have students add and subtract fractions with different or "unlike" denominators, Wilson handed his students inexpensive fractions calculators. These $12 calculators allowed his students to perform basic operations on fractions, simplify the answers, and convert fractions to decimals instantaneously. The calculator freed Leona from the laborious and often frustrating process of working these problems by hand. They allowed her to overcome causal, but catastrophic, errors that got in the way of understanding what she was doing. By using the tool for computations, Leona and her classmates were able to spend more time trying to understand what was special about fractions and what was actually happening when you multiplied or divided them.

Figure 1, page 75, shows the kind of worksheet that Leona completed as a paired activity with Katie, a classmate. They began by drawing the two fractions using number lines. Some students used pie charts, and others tried to estimate the answer using a ruler. Wilson briefly discussed the different answers, focusing on student solutions that involved some kind of mathematical reasoning. He also used the discussion as a vehicle for emphasizing why you can't add or subtract fractions with unlike denominators.

The next step in the exercise was for students to use their calculators to compute the correct answer. In previous years, computing the answer was the only point of the exercise. But with this lesson, students worked backwards to see the logic of converting fractions to common denominators before subtracting them. This also helped them understand why the answer generated by the calculator made sense. In fact, Wilson and other teachers in the pilot project consistently noted that the use of a variety of pictorial representations in conjunction with symbols (e.g., 1/4, 3/8) helped stu-

Figure 1. Worksheet: Calculators and Computations

$\frac{1}{2} \times \left(\frac{\ }{\ }\right) = \frac{\ }{\ }$ $\frac{1}{3} \times \left(\frac{\ }{\ }\right) = \frac{\ }{\ }$	**Step 1.** Teachers develop worksheets involving complex fractions such as the one to the left.
	**Step 2.** Students estimate their answers using manipulatives, drawings, or number lines. This involves a class discussion where different interpretations are discussed.
	**Step 3.** Students solve the problem using their fraction calculators
$\frac{1}{2} \times \left(\frac{\ }{\ }\right) = \frac{[illegible]}{6}$ $\frac{1}{3} \times \left(\frac{\ }{\ }\right) = \frac{[illegible]}{6}$ $\frac{1}{6}$	**Step 4.** Students write the final answer on the worksheet Since the answer is in sixths, the common denominator for both fractions in the problem should be in sixths
$\frac{1}{2} \times \left(\frac{3}{3}\right) = \frac{3}{6}$ $\frac{1}{3} \times \left(\frac{2}{2}\right) = \frac{2}{6}$ $\frac{1}{6}$	**Step 5.** Students write fractions equal to one in order to complete the problem. They also check the final answer to see that it makes sense.

dents understand difficult concepts like fractions. The students had come to realize that symbols alone were insufficient for remedial students and those with learning disabilities. Leona and Katie eventually worked 10 more of these problems for the remainder of the period while their teacher worked with different pairs of students.

Technology and Problem-Solving

For over a decade, the National Council of Teachers of Mathematics (NCTM) has criticized mathematics textbooks for asking students to solve artificial word problems (see also the box "Research Supporting New Approaches to Mathematics"). Wilson had always agreed with this point of view; but he, like many other teachers in charge of mainstreamed remedial classes, wasn't sure how to go beyond the textbook. For Wilson, asking students, "If the egg man had 500 eggs and he broke 1/4 of them unloading them from the truck, how many eggs did he break?" wasn't much different from, "What is 1/4 of 500?" These problems were little more than indirect computational practice. Too often he saw his students quickly look for keywords or just the numbers in the problem and rush to an answer with little thought about the problem itself. Students saw little purpose in the exercises, and problems in the world of work didn't come packaged in this way.

Through his involvement in the project, Wilson was able to put aside the textbook word problems and use the school's computer lab as an integral part of problem-solving and written communication. He was able to pose problems in class that would take a day or two to complete. After setting up the problem and having students gather data, they could spend the next day in the lab using Microsoft Excel, analyze the data, and present the results. Microsoft Word was useful for memos and other brief reports. Figure 2 shows a problem that Wilson asked his students to work on near the end of the year, after they had studied decimals, percents, and ratios. He asked six students in each of his two Math Apps classes to name their favorite musical groups. The next day he presented the results to the classes and asked the students to represent the data in terms of fractions, decimals, percents, and visually as a pie chart. Students worked in the lab in pairs, making sure that they had completed all aspects of the assignment.

When Wilson reviewed their printouts at the end of the period, he asked the students about the numbers and the charts, pointing out the common features of the different concepts. More important, he asked what the data might look like if he had asked every student in Marshall Junior High to name his or her favorite rock group:

> What if the 12 students that I asked were representative of all Marshall students? There are 720 students in the school. How

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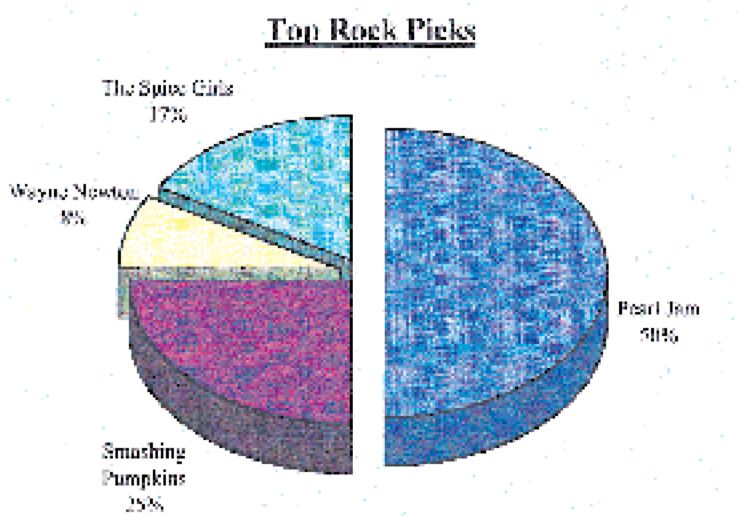
Figure 2. Rock Groups: Fractions, Decimals, and Percents in Context

	A	B	C	D	E	F
1						
2	rock group	formula	fraction	decimal	percent	
3						
4	Pearl Jam	=6/12	1/2	0.50	50%	
5	Smashing Pumpkins	=3/12	1/4	0.25	25%	
6	Wayne Newton	=1/12	1/12	0.08	8%	
7	The Spice Girls	=2/12	1/6	0.17	17%	
8						
9	Total	=12/12	1	1.00	100%	



	A	B	C	D	E
31					
32	Pearl Jam	50%			
33	Smashing Pumpkins	25%			
34	Wayne Newton	8%			
35	The Spice Girls	7%			
36					
37					
38					
39					
40					
41					

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many of them would like Pearl Jam? How many would like Smashing Pumpkins? If we sold Smashing Pumpkins CDs for $14.95 before school as part of a fund raiser, how much money would we raise if just 1/5 of our students bought them? And what would the profit be if we could buy the CDs for $12.25 wholesale? How would we figure this out based on the information that we've already collected?

Research Supporting New Approaches to Mathematics

The Workplace Literacy Project draws on recent middle and secondary school research that supports more *interactive teaching,* rather than traditional, direct instruction. An integration of mathematics, technology, and writing is particularly well-suited to instruction that emphasizes oral and written communication (Anders & Prichard, 1993; Englert, Raphael, & Anderson, 1992; Palincsar, Anderson, & David, 1993), as well as *mathematical problem-solving* (Bottge & Hasselbring, 1993; Woodward, Baxter, & Scheel, 1997). This kind of interactive instructional model encourages teachers to help students mediate their problem solving by first *scaffolding* techniques for simplifying complex problems:

- Problems that appear in everyday contexts.
- Problems that do not have "keywords."
- Problems that may take an instructional period or more to complete.

This kind of mediation is critical for students with learning disabilities, who often solve problems impulsively (Cawley, 1985; Montague, 1992).

Recent mathematics research also supports the use of *multiple representations* and the use of *discourse* to link those different representations as an effective means of developing conceptual understanding (see Hiebert & Carpenter, 1992). By emphasizing *conceptual understanding* and *limiting procedural knowledge* (e.g., multiplying simple two-digit by one-digit decimals manually), students develop a mathematical framework within which they can use calculators and number sense to solve and explain *authentic problems.*

This process was confirmed in a recent study (Woodward, Baxter, & Robinson, in press) where researchers used an array of visual representations (e.g., fraction bars, decimal squares, overheads) as teaching tools and encouraged student discussion. Calculators were an integral part of classroom exercises, as follows:

- As a tool for promoting the linkage between concepts (e.g., as a means of demonstrating that 1/4 was equivalent to 25/100 or .25) .
- As a part of problem-solving and checking answers.

Results of the study showed that these students had greater conceptual understanding than did students in the comparison group, who learned about decimals and percents through a sophisticated direct instruction approach to the topics.

Wilson was pushing students to do several important activities in this exercise. First, he was having them work from data and look at it from several points of view. Even though students readily understood one aspect of the problem (i.e., musical groups), they were being asked to think about it formally and systematically. How could a small sample of students accurately represent a larger group? Yet this idea of collecting and analyzing data as a math activity has been at the heart of math reform proposed by organizations like NCTM.

Second, Wilson was subtly pushing students to use ratios as a way to solve the problem. Again, the context made it easier for students to see how a complex concept like ratios actually had a "real life" utility. Wilson was also able to extend the problem into a businesslike scenario, one that availed itself to modeling on the spreadsheet. In fact, the idea of working with data and asking different questions about it reinforced the value of the spreadsheet as a mathematical tool. Through carefully designed problems, Wilson was able to integrate technology with problems that were grounded in the students' lives.

Technology and the World of Work

Workforce Literacy problems at the high school level attempt to draw a closer connection between ones developed at school and ones in the world of work. A good example of relationship is the "french fries problem" that Roberta Johnson used in her 11th-grade class. Building on the predictable dietary habits of most teenagers, Johnson constructed a problem that could extend from the students' world to an aspect of work that her students had contact with on a regular basis: McDonald's.

Johnson first asked students to look closely at the servings of french fries in the school cafeteria. After a day's observation, her students noted considerable variation in the number of fries from one basket to the next. After that day's classroom discussion, students then systematically investigated the problem by purchasing, weighing, counting, measuring, and eating french fries for a week. The students compiled the data on a spreadsheet file; and at the end of the week, the class analyzed the results. By calculating the average length of their french fries, students could quickly see that baskets with greater numbers of fries had more fries that were shorter than average. The converse was also true. Two line graphs—one showing the total number of fries in a basket and the other showing the number of fries below the average—also demonstrated this relationship when weight of the basket was held constant. Yet the weight of each basket also varied, to the point that students were surprised at how many more fries some students would get than others. Johnson's students used Microsoft Word to report this finding in the form of a memo to the cafeteria cooks and recommend two different solutions to the problem.

Following the analysis of school french fries, Roberta Johnson extended this problem to the McDonald's scenario. She first asked the students what they thought would happen if McDonald's added "just a couple of fries" to each large basket of french fries. After all, what difference could a couple of french fries make? All of the students agreed that this would make very little difference overall. Johnson then helped them analyze the problem in greater detail so that they could model it on the spreadsheet. By assuming that there were about 75 fries in each large basket, they found that the implications were enormous. Adding just 3 french fries to a large basket meant that the company would be giving away every 16th basket of fries for free. At $1.09 per basket, their spreadsheet model indicated that this could be as much as a $25,000 loss in revenues per day. Johnson concluded this exercise by showing how the same concern for precise, consistent measurement was also true in stores as diverse as Starbucks coffee and Baskin Robbins ice cream.

Practice on isolated skills does little to prepare students with disabilities for the work force.

Tools for Understanding (http://www.ups.edu/community/tofu) offers teachers a foundation for helping students achieve the kind of literacy that they will need in the future world of work.

Tools for Understanding

Funding for the Workforce Literacy Project ended in 1997. The project, however, has culminated in a supplementary curriculum that is available free on the Internet: Tools for Understanding (http://www.ups.edu/community/tofu). Tools for Understanding offers secondary teachers a variety of ways to go beyond traditional methods for teaching math. The curriculum presents model lessons for exploring common math topics with remedial and special education students. It offers detailed suggestions on how to balance paper-and-pencil practice with conceptual approaches that make use of pictorial representations and common technologies. It also describes how students can incorporate various types of writing—from daily math journals to formal memos—into daily lessons. Tools for Understanding offers teachers a foundation for helping students achieve the kind of literacy that they will need in the future world of work.

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